Supplemental material for: Bound states in the continuum through environmental design

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(Dated: April 4, 2019)

I. PROPERTIES OF THE OUTCOUPLING COEFFICIENT ACROSS LINES OF BICS

In the main text, two statements are made about the properties of the outcoupling coefficient, $d_0(\mathbf{k}_\parallel) = |d_0(\mathbf{k}_\parallel|e^{i\theta(\mathbf{k}_\parallel)}$, as $\mathbf{k}_\parallel$ is continuously varied such that it crosses a line of BICs in the single radiation channel regime. First, as BICs represent a topological defect the phase of the outcoupling coefficient, $\theta(\mathbf{k}_\parallel$, jumps by π across the line of BICs. And second, that it is possible to choose a gauge for the outcoupling coefficient such that $\tilde{d}_0 \in \mathbb{R}$ and that $\tilde{d}_0$ changes sign across the line of BICs, proving that the photonic crystal slab possesses a BIC and not simply a high-Q resonance. Here, we provide evidence for those statements.

In Fig. S1a we show the numerically calculated phase of d_0, $\theta(\mathbf{k}_\parallel)$, and observe the phase jump of d_0 across the line of BICs. To do so, we note that it is possible to directly calculate D using the eigenmode decomposition feature of Meep [1]. This is consistent with the phase jump of π observed across lines of BICs found in layers of stacked birefringent materials [2], and demonstrates that the line of BICs is a 1D topological defect, similar to domain walls in spin-up/down systems.

To demonstrate that the systems studied in the main text achieve true BICs and not simply high-Q resonances, we first note that when the system is C_2 symmetric, and thus obeys to Eq. 3, the phase of $D(\mathbf{k}_\parallel)$ is always defined relative to $C(\mathbf{k}_\parallel)$, the direct scattering coefficients between the incoming and outgoing signals. Thus, in the single radiation channel regime, using Eq. 4 one can choose the gauge of d_0 at every $\mathbf{k}_\parallel$ such that $d_0 = \tilde{d}_0 \in \mathbb{R}$. By further requiring that this choice of gauge is smooth as a function of $\mathbf{k}_\parallel$, one can see in Fig. S1b that $\tilde{d}_0$ changes sign across the line of BICs, proving that $D(\mathbf{k}_\parallel) = 0$ for some choice of $\mathbf{k}_\parallel$.

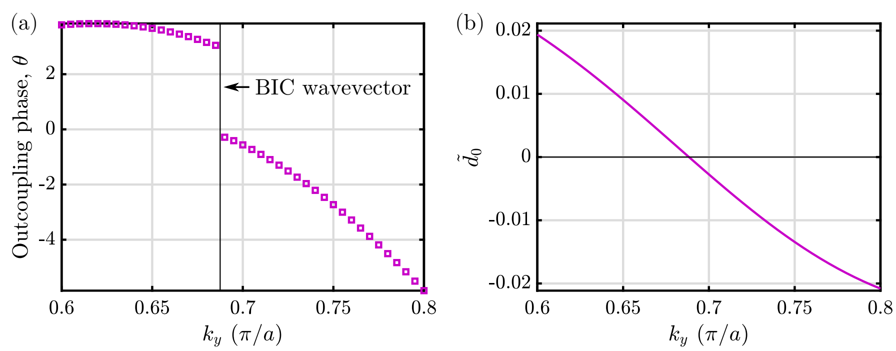

FIG. S1. (a) Plot of the phase of the outcoupling coefficient, $\theta(\mathbf{k}_\parallel)$, with fixed $k_x = 0.7(\pi/a)$ and k_y varied so as to cross the line of BICs for the C_{2v} symmetric system shown in Fig. 1 of the main text. (b) Plot of the real outcoupling coefficient $\tilde{d}_0(\mathbf{k}_\parallel)$ for the same choice of wavevectors and system.

[1] Ardavan F. Oskooi, David Roundy, Mihai Ibanescu, Peter Bermel, J. D. Joannopoulos, and Steven G. Johnson, "Meep: A flexible free-software package for electromagnetic simulations by the FDTD method," Comput. Phys. Commun. **181**, 687–702 (2010).

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[2] Samyobrata Mukherjee, Jordi Gomis-Bresco, Pilar Pujol-Closa, David Artigas, and Lluis Torner, "Topological properties of bound states in the continuum in geometries with broken anisotropy symmetry," Phys. Rev. A **98**, 063826 (2018).