Achieving Arbitrary Control over Pairs of Polarization States Using Complex Birefringent Metamaterials

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We demonstrate that the key to realizing arbitrary control over pairs of polarization states of light, i.e., transforming an arbitrarily polarized pair of input states to an arbitrarily polarized pair of output states, is the ability to generate pairs of states with orthogonal polarizations from nonorthogonal pairs of initial states. Then, we develop a new class of non-Hermitian metamaterials, termed complex birefringent metamaterials, which are able to do exactly this. Such materials could facilitate the detection of small polarization changes in scattering experiments as well as enable new polarization multiplexing schemes in communications networks.

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Polarization is one of the fundamental properties of light, and control over the polarization is paramount in many optical communications and imaging applications. In general, the effect of propagation through any media on the polarization of an incident electromagnetic signal can be described as $|\beta\rangle = S(z)|\alpha\rangle$, where $|\alpha\rangle$ and $|\beta\rangle$ are the input and output polarization states, respectively, and $S(z)$ is a 2×2 matrix that depends on the properties of the medium, as well as the propagation distance z. Conventionally, the polarization of a signal is manipulated through the use of birefringent materials [1–9]. For lossless birefringent media, with proper choice of material parameters and propagation distance, it is always possible to convert an input polarization $|\alpha_1\rangle$ to an arbitrary output polarization $|\beta_1\rangle$. However, once the response to $|\alpha_1\rangle$ is determined, the output polarization $|\beta_2\rangle = S|\alpha_2\rangle$ is no longer arbitrary for any other input polarization $|\alpha_2\rangle$. This is because S is unitary in lossless media, and thus, $\langle\beta_2|\beta_1\rangle = \langle\alpha_2|\alpha_1\rangle$.

In this Letter, we seek to overcome the limitation of conventional birefringent media by developing a class of metamaterials which enable arbitrary control over pairs of polarization states. By arbitrary control, we demand that, for a pair of arbitrary input polarizations $|\alpha_1\rangle$ and $|\alpha_2\rangle$, one can generate an arbitrary pair of output polarizations $|\beta_1\rangle$ and $|\beta_2\rangle$. Achieving such polarization control has significant implications for a wide range of technologies. For example, with this capability, one can map two polarizations that are close to each other into two orthogonal polarizations, which may facilitate the detection of small polarization changes, such as those arising from the imaging of biological tissues [10,11] and thin films [12]. Likewise, the ability to completely separate nonorthogonal polarization states could enable new multiplexing schemes in optical communications networks beyond what is currently possible [13,14].

First, we show that in order to achieve arbitrary control over pairs of polarization states, it is sufficient to develop a class of metamaterials which are capable of performing the following polarization transformation as denoted by S_θ:

$$|1,1\rangle = S_\theta|\theta\rangle, \tag{1}$$

$$|1,-1\rangle = S_\theta|-\theta\rangle. \tag{2}$$

Here, we assume propagation along the z axis, and label the polarization states in terms of the electric field components in the xy plane as $|E_x, E_y\rangle$. $|\pm\theta\rangle$ denote the two polarization states that lie on the great circle of the Poincaré sphere passing through $|1,i\rangle$ and $|1,1\rangle$, and are symmetrically placed away from $|1,i\rangle$, subtending an angle of $\pm\theta$ with respect to $|1,i\rangle$. Suppose we can construct a class of materials which can provide S_θ for an arbitrary θ. For an arbitrary pair of input states $|\alpha_1\rangle$ and $|\alpha_2\rangle$, using conventional lossless birefringent materials, one can achieve the transformation [4,5,8]

$$|\theta_\alpha\rangle = U_\alpha|\alpha_1\rangle, \tag{3}$$

$$|-\theta_\alpha\rangle = U_\alpha|\alpha_2\rangle, \tag{4}$$

where U_α is unitary and $\langle\theta_\alpha|-\theta_\alpha\rangle = \langle\alpha_1|\alpha_2\rangle$. For the pair of arbitrary output states $|\beta_1\rangle$ and $|\beta_2\rangle$, one can obtain a similar unitary transformation U_β that transforms them to $|\pm\theta_\beta\rangle$. Therefore, the transformation S from the input states $|\alpha_1\rangle$ and $|\alpha_2\rangle$ to the output states $|\beta_1\rangle$ and $|\beta_2\rangle$ is then

$$S = U_\beta^\dagger S_{\theta_\beta}^{-1} S_{\theta_\alpha} U_\alpha. \tag{5}$$

To achieve the transformation as described by Eqs. (1) and (2) in general requires a non-Hermitian metamaterial. Now, we proceed to show that such a transformation can be realized in a class of complex symmetric metamaterials with its dielectric tensor having the form

$$\bar{\varepsilon} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix} \equiv \begin{pmatrix} \bar{\varepsilon}_\perp & 0 \\ 0 & \varepsilon_{zz} \end{pmatrix}, \tag{6}$$

in which

$$\varepsilon_{xx} = \varepsilon_{yy}^* = \varepsilon_r - i\varepsilon_i, \tag{7}$$

$$\varepsilon_{xy} = \varepsilon_{yx}, \tag{8}$$

where ε_r, ε_i, $\varepsilon_{xy} \in \mathbb{R}$. Here, an equal amount of gain and loss has been added to the x and y axes of a conventional birefringent material, a choice inspired by recent developments in optical media with spatially distributed regions containing equal amounts of gain and loss [15–30]. Henceforth, we refer to materials which obey Eqs. (7) and (8) as complex birefringent metamaterials.

For light propagating along the z axis of such a medium, the allowed wave vectors, $k_\pm$, of a monochromatic signal with frequency ω, can be found by solving the right eigenvalue equation [31,32],

$$\omega^2\mu\bar{\varepsilon}_\perp|E_\pm^R\rangle = k^2|E_\pm^R\rangle, \tag{9}$$

in which μ is the scalar magnetic permeability. As such, the allowed wave vectors and right eigenpolarizations of complex birefringent metamaterials are

$$\frac{k_\pm^2}{\omega^2\mu} = \varepsilon_r \pm \varepsilon_{xy}\sqrt{1-\tau^2}, \tag{10}$$

$$|E_\pm^R\rangle = \frac{1}{N_\pm}|1, i\tau \pm \sqrt{1-\tau^2}\rangle, \tag{11}$$

in which $\tau = \varepsilon_i/\varepsilon_{xy}$ represents a normalized measure of the strength of the gain and loss in the system, and

$$N_\pm^2 = 2(1-\tau^2) \pm 2i\tau\sqrt{1-\tau^2}, \tag{12}$$

is the normalization of the eigenstates. The matrix $\omega^2\mu\bar{\varepsilon}_\perp$ also has left eigenpolarizations, which are solutions to $\langle E_\pm^L|\omega^2\mu\bar{\varepsilon}_\perp = k_\pm^2\langle E_\pm^L|$. Together, the left and right eigenpolarizations form a biorthogonal basis, and can be normalized such that $\langle E_m^L|E_n^R\rangle = \delta_{mn}$, with the choice of $N_\pm$ in Eq. (12).

In conventional lossless birefringent media, $\bar{\varepsilon}_\perp$ is Hermitian, and so $\langle E_\pm^L| = |E_\pm^R\rangle^\dagger$. However, for complex birefringent materials, $\omega^2\mu\bar{\varepsilon}_\perp$ is complex-symmetric, and the left and right eigenpolarizations are related by $\langle E_\pm^L| = |E_\pm^R\rangle^T$. Moreover, although the two right eigenpolarizations are linearly independent for complex birefringent materials with $|\tau| \neq 1$, it can be readily seen that they are not orthogonal, $\langle E_\mp^R|E_\pm^R\rangle \neq 0$ except for when $\tau = 0$ and the system reverts to a conventional birefringent material, or when $|\tau| \to \infty$ and the system becomes a conventional dichroic material.

The evolution of the polarization of light propagating within a complex birefringent material with $|\tau| \neq 1$ can be expressed in terms of the right eigenpolarizations (11), as

$$|E(z)\rangle = e^{ik_- z}(e^{i\Delta k z}A_+|E_+^R\rangle + A_-|E_-^R\rangle), \tag{13}$$

in which $\Delta k = k_+ - k_-$ is the additional phase accumulated by $|E_+^R\rangle$ relative to $|E_-^R\rangle$ per unit length, and the initial amplitudes, $A_\pm$, are defined in terms of the left eigenpolarizations as $A_\pm = \langle E_\pm^L|E(0)\rangle$. The resulting polarization dynamics of a complex birefringent material can be visualized by plotting the output polarization as a function of z on the Poincaré sphere. The example shown in Fig. 1(a) illustrates the polarization dynamics when $|\tau| < 1$, for which both allowed wave vectors are real, $k_\pm \in \mathbb{R}$. This is analogous to the exact phase in parity-time symmetric systems [17–19]. For the sake of comparison, we also plot the polarization dynamics for a conventional Hermitian birefringent material in Fig. 1(b), as described by setting $\varepsilon_i = 0$ in Eq. (6). For both types of materials, as shown in Fig. 1, the eigenpolarizations define the fixed point of the dynamics. Thus, so long as the incident polarization is not parallel to one of these eigenpolarizations, the polarization of the initial signal forms a closed trajectory around the eigenpolarizations as z is varied.

When $\tau = 0$, which describes a conventional lossless birefringent material, the two eigenpolarizations are located at $|1,1\rangle$ and $|1,-1\rangle$, corresponding to two linearly polarized states. The two eigenstates are orthogonal to each other, and the polarization trajectories form circles around the axis formed by the two eigenstates, shown in Fig. 1(b).

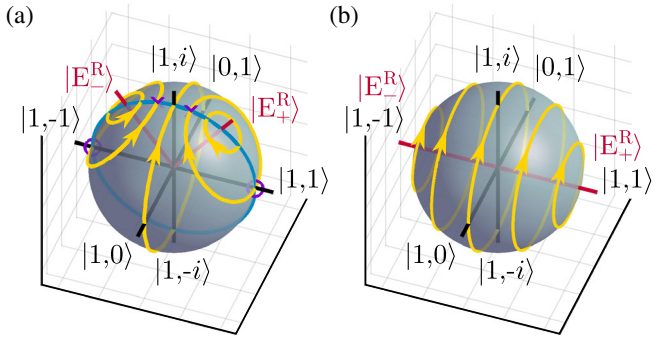

FIG. 1. (a) Flow lines (yellow) depict the polarization dynamics on the Poincaré sphere when light travels along the z direction through a complex birefringent material as described by Eqs. (6)–(8) with $\tau = 0.81$. Note that the shape of the flow lines depends only the value of τ. The great circle containing $|1,i\rangle$ and $|1,1\rangle$ is shown in blue. The nonorthogonal eigenpolarizations of the dielectric tensor which form the axis of polarization flow are shown in red. The purple crosses show the initial nonorthogonal polarization states $|\pm\theta\rangle$ which can be mapped to the orthogonal polarization states $|1,\pm1\rangle$, shown as purple circles, as described by Eqs. (1) and (2). (b) Flow lines (yellow) depict the polarization dynamics on the Poincaré sphere when traveling through a conventional birefringent material, with $\tau = 0$. The orthogonal eigenpolarizations of the dielectric tensor are shown in red.

(On the Poincaré sphere, orthogonal states are represented by antipodal points.) As τ is increased, so that $0 < \tau < 1$, the two eigenpolarizations remain on the great circle connecting $|1,1\rangle$ and $|1,i\rangle$, but are tilted away from the states $|1,\pm1\rangle$ towards $|1,i\rangle$, which is a manifestation of the nonorthogonality of these two eigenstates. For $-1 < \tau < 0$, the eigenstates tilt away from $|1,\pm1\rangle$ towards $|1,-i\rangle$. In both cases, as $k_\pm \in \mathbb{R}$, the polarization trajectories are still closed, but are no longer centered on the axis formed by the eigenstates, as shown in Fig. 1(a).

Examining the polarization dynamics of Fig. 1(a), we note that there are two states lying on the great circle connecting $|1,1\rangle$ and $|1,i\rangle$, indicated by the purple crosses, which can be transformed to the two states $|1,1\rangle$ and $|1,-1\rangle$, indicated by purple circles, with a proper choice of the system parameters. Therefore, complex birefringent metamaterials with $0 < |\tau| < 1$, indeed, provide the key nontrivial step required for achieving arbitrary control over pairs of polarization states, which is to realize S_θ as defined in Eqs. (1) and (2). Mathematically, the complex birefringent metamaterial which transforms a given pair of input states $|\pm\theta\rangle$ to the two final states $|1,\pm1\rangle$ satisfies

$$e^{i\Delta kl} = \frac{\langle E_-^L|\pm\theta\rangle\langle E_+^L|1,\pm1\rangle}{\langle E_+^L|\pm\theta\rangle\langle E_-^L|1,\pm1\rangle}, \qquad |\tau| < 1. \tag{14}$$

Equation (14) represents a single complex transcendental equation, as τ appears in both Δk and the left eigenstates $\langle E_\pm^L|$, and solving Eq. (14) for either choice of $\pm$ yields a solution which satisfies the other choice. In practice, there are four independent properties of the system which can be tuned to satisfy this criteria, ε_r, ε_i, ε_{xy}, and l, and yet, Eq. (14), as a complex equation, provides only two constraints; therefore, many different systems can be found which realize the same transformation S_θ. As such, for a fixed l, one can modify the dielectric parameters to achieve different S_θ. Thus, a tunable metamaterial [33–36] with a fixed length can be used to achieve arbitrary polarization control. In general, as the choice of initial states become parallel ($\theta \to 0$), stronger effective gain and loss, $|\tau| \to 1$, as well as longer effective propagation distances, Δkl, are required to produce orthogonal output states.

Up to this point, we have focused on the regime of $|\tau| < 1$; however, this class of metamaterials also exhibits interesting polarization dynamics with $|\tau| \geq 1$. When $|\tau| > 1$, both eigenvalues $k_\pm$ become complex, with $k_+ = k_-^*$. This is analogous to the broken phase in parity-time symmetric systems. In this regime, the eigenpolarizations reside along the great circle on the Poincaré sphere connecting $|1,0\rangle$, $|0,1\rangle$, and $|1,\pm i\rangle$. As z varies, the eigenstates correspond to a stable or an unstable fixed point on the Poincaré sphere depending on the sign of $\mathrm{Im}[k_\pm]$, as shown in Fig. 2(a), and the material provides polarization-dependent attenuation and amplification. As $|\tau| \to \infty$, the system becomes a conventional dichroic material with orthogonal eigenpolarizations $|1,0\rangle$ and $|0,1\rangle$.

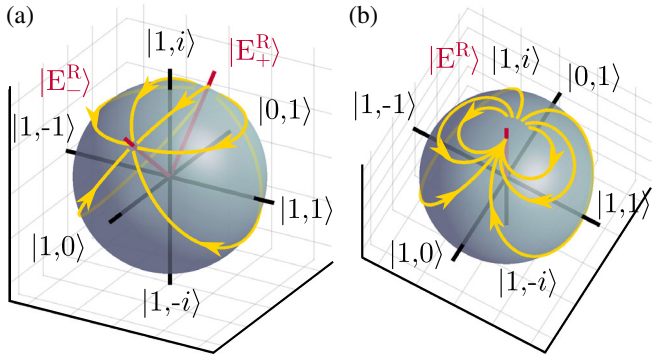

FIG. 2. (a) Flow lines (yellow) depict the polarization dynamics on the Poincaré sphere when light travels along the z direction through a complex birefringent material as described by Eqs. (6)–(8) with $\tau = 1.5$. The eigenvectors of the dielectric tensor are shown in red, with the arrow indicating the direction in which the polarization flows for $\varepsilon_{xy} > 0$. (b) Flow lines (yellow) depict the polarization dynamics when light travels along the z direction through a complex birefringent material as described by Eqs. (6)–(8) with $\tau = 1$. The single eigenpolarization of the system is $|1,i\rangle$ (red).

When $|\tau| = 1$, complex birefringent materials possess an exceptional point [37,38] where both the eigenvalues and eigenvectors coalesce, and the eigenvectors become self-orthogonal, $\langle E_\pm^L|E_\pm^R\rangle = 0$. This yields two unique properties. First, the expression for the evolution of the electric field (13) is no longer valid as $\omega^2\mu\bar{\varepsilon}_\perp$ has a nontrivial Jordan normal form. Instead, the evolution of the field must be expressed in terms of the single remaining eigenvector, $|E^R\rangle$ and its associated Jordan vector, $|J^R\rangle$ [39,40], as

$$|E(z)\rangle = e^{ik_0 z}[(A_E + iA_J k_0 z)|E^R\rangle + A_J|J^R\rangle], \tag{15}$$

in which $k_0 = \omega\sqrt{\mu\varepsilon_r}$, and A_E, A_J are the modal amplitudes at $z = 0$. The derivation of this equation is provided in the Supplemental Material [41]. As can be seen in Eq. (15), the polarization of light flowing through a complex birefringent material at $|\tau| = 1$ has only a single fixed point that corresponds to circularly polarized light, to which the polarization of every initial state with $A_J \neq 0$ converges through linear amplification as a function of z. Second, any initial polarization state converges to the fixed point from a single direction, along the $|1,0\rangle$ to $|1,i\rangle$ contour for $\tau = 1$ as shown in Fig. 2(b). This is distinct from what is observed for the stable fixed point when $|\tau| > 1$, in which $|E_-^R\rangle$ can be approached from any direction.

In all three of their phases with $\tau \neq 0$, complex birefringent materials are necessarily active optical structures and, in general, do not conserve the intensity of the incident radiation. This fact is immediately evident from the linear and exponential amplification present when $|\tau| \geq 1$. When $|\tau| < 1$, $k_\pm$ are real, and the intensity is a periodic function of z,

$$I(z) = |A_+|^2\langle E_+^R|E_+^R\rangle + A_+^* A_- e^{-i\Delta kz}\langle E_+^R|E_-^R\rangle + |A_-|^2\langle E_-^R|E_-^R\rangle + A_-^* A_+ e^{i\Delta kz}\langle E_-^R|E_+^R\rangle. \quad (16)$$

Thus, in this case, the total change in the intensity is bounded. Even though the intensity is not conserved in complex birefringent materials, two generalized unitarity relations can be derived which relate the transmission and reflection coefficients of the scattering matrix, S [23,42], as shown in the Supplemental Material [41]. These conserved quantities stem from the fact that complex birefringent materials are invariant upon the operation that switches the x and y axes of the system, $\mathcal{M}$, and the time-reversal operation, $\mathcal{T}$, thus,

$$(\mathcal{MT})S(\mathcal{MT}) = S^{-1}. \quad (17)$$

There are many possible experimental realizations of complex birefringent materials. A dielectric response as described by Eqs. (6)–(8) has been previously realized experimentally in a metasurface structure [28]. However, in order to observe the polarization dynamics effects and to achieve the capability for arbitrary control over pairs of polarization states, neither of which are considered in [28], it would be interesting to create three-dimensional media where the effective propagation distance can be varied. Therefore, here we focus on the construction of three-dimensional systems with the appropriate dielectric tensor. As an example, one could construct a metamaterial consisting of an ordinary birefringent material interspersed with layers containing regions of both gain and loss, as depicted in Fig. 3(a). Transfer matrix calculations of this exact structure without using the effective medium approximation in the propagation direction confirm its ability to separate a pair of initial states with similar polarizations to be nearly orthogonal, shown in Fig. 3(b). A tunable version of this system is discussed in the Supplemental Material [41].

Alternatively, there are many methods for adding birefringence to optical fibers geometrically, allowing for $\varepsilon_{xy} \neq 0$. By doping such a birefringent fiber, gain could be added to both ε_{xx} and ε_{yy}. Then, all that is required to realize complex birefringence is the ability to add loss specifically to ε_{yy}. Regardless of the specific realization chosen, the experimental design of complex birefringent materials benefits from the critical feature that the amount of gain and loss, ε_i, necessary to observe significant nontrivial polarization dynamics is set by the anisotropy of the system, ε_{xy}, which can be designed to be quite small. Thus, very little gain or loss is necessary to realize arbitrary control over pairs of polarization states in these materials.

Here, we have focused on metamaterials with $\varepsilon_{xx} = \varepsilon_{yy}^*$, as this choice yields a regime of parameter space, $|\tau| < 1$, where the eigenvalues of the system are real, and as such, the change in intensity of an incident signal is bounded. However, many other choices of $\varepsilon_{xx}, \varepsilon_{yy} \in \mathbb{C}$, such as

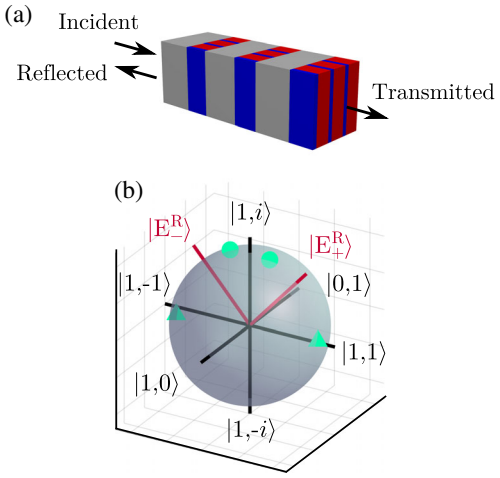

FIG. 3. (a) Schematic of a complex birefringent metamaterial, consisting of layers of a conventional birefringent material (gray), and layers of a material containing gain (red) and loss (blue), forming a comb. The patterning of the structure is assumed to be fine enough relative to the wavelength of the light so as to be in the effective medium limit. (b) Transformation of the polarization on the Poincaré sphere of two signals through 59 μm of a complex birefringent metamaterial, as shown schematically in (a). Here, for an incident light with wavelength 1.55 μm, we have used calcium carbonate whose ordinary and extraordinary axes are rotated 7.47° with respect to the lab frame, yielding a dielectric tensor with $\varepsilon_{xx} = 2.66$, $\varepsilon_{yy} = 2.19$, $\varepsilon_{xy} = 0.063$. The isotropic gain has $\varepsilon = 4 - 0.1i$, and the isotropic loss has $\varepsilon = 9.74 + 0.63i$. By forming a comb consisting of 77% gain regions and 23% loss regions, the effective dielectric tensor is anisotropic in this layer, with $\varepsilon_{\|} = 5.34 + 0.07i$ and $\varepsilon_{\|} = 4.64 - 0.07i$. By using 30 nm layers of calcium carbonate, and 20 nm layers of the gain and loss, the total system constitutes a complex birefringent metamaterial with $\varepsilon_{xx} = 3.454 - 0.028i$, $\varepsilon_{yy} = 3.453 + 0.028i$, and $\varepsilon_{xy} = 0.038$ (see Supplemental Material [41]), which corresponds to $\tau = 0.75$. The initial signal polarizations are separated by 16.2° (cyan circles), while the final polarization states are nearly orthogonal (cyan triangles). The surrounding medium is air, $\varepsilon = 1$.

birefringent materials with loss in a single polarization channel, i.e., $\varepsilon_{xx} = \varepsilon_r$ and $\varepsilon_{yy} = \varepsilon_r + 2i\varepsilon_i$ with $\varepsilon_{xy} \neq 0$, will still yield nonorthogonal eigenvectors, which can allow for nearly arbitrary control over pairs of polarization states as shown in Fig. S2 in the Supplemental Material [41]. Similarly, off-axis propagation in directions which do not conserve $\mathcal{M}$ also results in an entirely complex spectrum even for $|\tau| < 1$. Fortunately, the rate at which $k_\pm$ acquires an imaginary component for off-axis propagation is slow relative to the change in propagation angle, as shown in Fig. S3 [41], and thus, the off-axis components of a wave packet traveling through a complex birefringent metamaterial will experience similar polarization dynamics to the on-axis component.

In conclusion, we have developed a theory of complex symmetric anisotropic dielectric materials and demonstrated that such systems enable arbitrary control over

pairs of polarization states. In particular, such complex birefringent materials may have applications in both splitting signals with adjacent polarizations and nearly combining signals with orthogonal polarizations.

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