Supplementary information for "Classifying topology in photonic crystal slabs with radiative environments"

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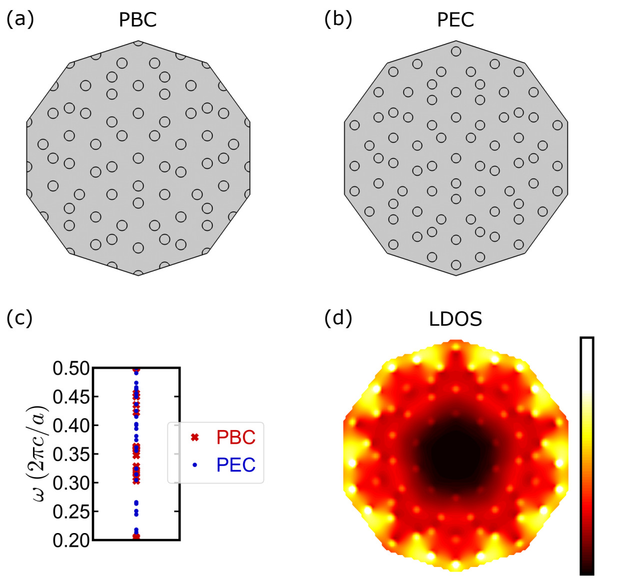

FIG. S1. **Spectra of the photonic quasicrystal.** Design of the photonic Chern quasicrystal based on a Penrose tiling, with (a) continuous periodic (PBC) and (b) perfect electric conductors (PEC) boundary conditions. The rhombuses composing the Penrose tiling have sides of length a, and the dielectric rods are located at the vertices of the Penrose tiling in a gyro-electric background, $\bar{\epsilon}_{jj} = 1$, $\bar{\epsilon}_{xy} = -0.4i$, with radius $r = 0.18a$. (c) Spectra of the photonic system along the real axis, with continuous periodic (PBC) and with perfect electric conductor (PEC) boundary conditions to simulate the system without and with edges, respectively. (d) Local density of states (LDOS) for the H_z component of the field at $\omega_0 = 0.37[2\pi c/a]$ with the geometry in panel (b).

Supplementary note 1. BULK AND EDGE SPECTRA OF THE PHOTONIC QUASICRYSTAL

This section show how the bulk and edge spectra of the quasicrystal can be calculated by applying appropriate boundary conditions [1]. Using continuous periodic boundary conditions (PBC), namely an equivalent Bloch phase with $\boldsymbol{k} = (0,0)$, on the opposite edges of the polygon [Fig. S1(a)], the quasicrystal is effectively repeated and does not have any edges. Using perfect electric conductor (PEC) boundary conditions on the edges of the polygon [Fig. S1(b)], the photonic system now has an edge and can be assumed as not being surrounded by a homogeneous material. The obtained spectra for both the PBC and PEC boundary conditions [Fig. S1(c)] therefore correspond to the spectra for the bulk and edge states, respectively. There are some frequency ranges which are empty for the PBC spectrum while having some eigenvalues for the PEC, demonstrating the presence forbidden frequency ranges for the bulk states and the presence of the edge states. Figure S1(d) plots the local density of states (LDOS) of the edge state at $\omega_0 = 0.37[2\pi c/a]$. Compared to the LDOS in the main text [Fig. 4(b)], the LDOS here is not obscured by the bulk states from the homogeneous surrounding.

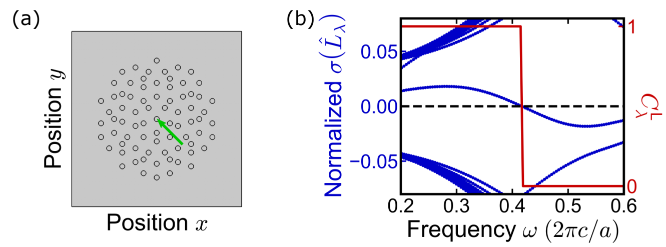

FIG. S2. **Probing of the local topology along the frequency axis in a photonic Chern quasicrystal system.** (a) Design of the photonic Chern quasicrystal based on a Penrose tiling. The rhombuses composing the Penrose tiling have sides of length a, and the dielectric rods are located at the vertices of the Penrose tiling in a gyro-electric background, $\bar{\epsilon}_{jj} = 1$, $\bar{\epsilon}_{xy} = -0.4i$, with radius $r = 0.18a$. (b) Spectrum of the FEM-localizer $\sigma\left(\hat{L}_{\boldsymbol{\lambda}=(x_0,y_0,\omega)}\right)$ normalized by $10^{-4}\|H_{\text{eff},c}(\omega_0)\|$ and the local Chern number $C^{\text{L}}_{\boldsymbol{\lambda}=(x_0,y_0,\omega)}$ along the ω axis at the center of the system $(x_0, y_0) = (0,0)$, as indicated by the green arrow in (a), with $\kappa = 1\left[10^{-4}\|H_{\text{eff},c}(\omega_0)\|/\|X_c\|\right]$ and $\omega_0 = 0.37[2\pi c/a]$.

Supplementary note 2. TOPOLOGICAL FREQUENCY RANGE IN THE QUASICRYSTAL

Here, we present the topology of the quasicrystal along some frequency range. The topology in the frequency axis is obtained from the spectrum of the localizer $\sigma\left(\hat{L}_{(x_0,y_0,\omega)}\right)$ along the ω axis at the center of the system $(x_0, y_0) = (0,0)$, as plotted in Fig. S2. The figure shows the change of topology via the crossing the spectrum of $\hat{L}$ with the zero axis at frequency $\omega = 0.42[2\pi c/a]$, matching the end of the forbidden frequency range seen

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in Fig. S1(c). Although the system is aperiodic and therefore lack of band structures, the spectral localizer is able to determine the frequency range for which the system exhibits non-trivial topology or not.

[1] Y. Zhang, Z. Lan, L. Hu, Y. Shu, X. Yuan, P. Guo, X. Peng, W. Chen, and J. Li, Chiral photonic topological states in Penrose quasicrystals, Optics Letters **48**, 2229 (2023), 2304.12802.